

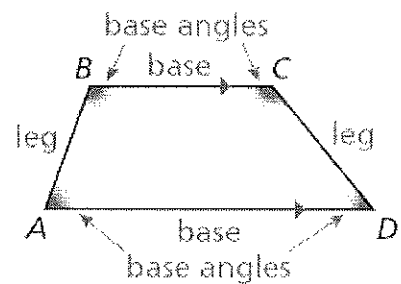
## Trapezoids and Kites

### Vocabulary:

#### Trapezoid

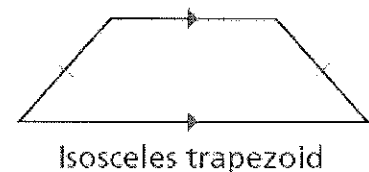
- a quadrilateral with exactly one pair of parallel sides
- the parallel sides are the **bases**
- the **base angles** of a trapezoid are two consecutive angles whose common side is a base
- a trapezoid has two pairs of base angles

In trapezoid ABCD,  $\angle A$  and  $\angle D$  are one pair of base angles, and  $\angle B$  and  $\angle C$  are the second pair.



The nonparallel sides are the **legs** of the trapezoid.

If the legs of a trapezoid are congruent, then the trapezoid is an **isosceles trapezoid**

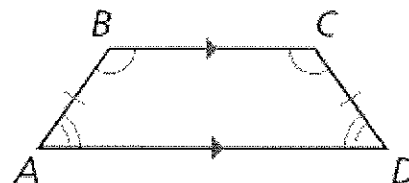


#### Isosceles Trapezoid Base Angles Theorem

- if a trapezoid is isosceles, then each pair of base angles is congruent

if trapezoid ABCD is isosceles

then  $\angle A \cong \angle D$  and  $\angle B \cong \angle C$

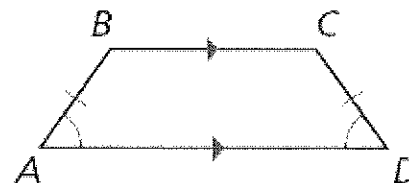


#### Isosceles Trapezoid Base Angles Converse

- if a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid

if  $\angle A \cong \angle D$  (or if  $\angle B \cong \angle C$ )

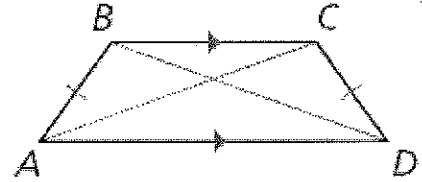
then trapezoid ABCD is isosceles



**Isosceles Trapezoid Diagonals Theorem**

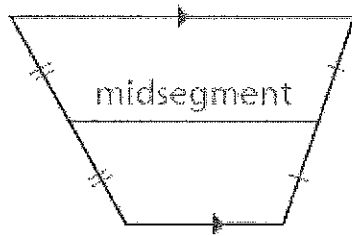
- a trapezoid is isosceles if and only if its diagonals are congruent

Trapezoid ABCD is isosceles if and only if  $\overline{AC} \cong \overline{BD}$



**Midsegment of a Trapezoid**

- the segment that connects the midpoints of its legs

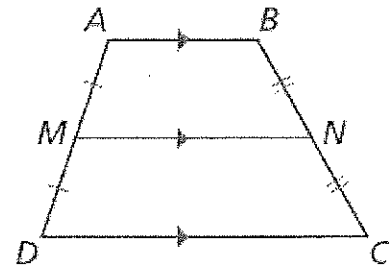


**Trapezoid Midsegment Theorem**

- the midsegment of a trapezoid is parallel to each base
- its length is one-half the sum of the lengths of the bases

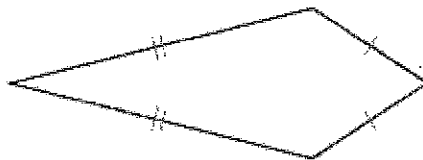
if  $\overline{MN}$  is the midsegment of trapezoid ABCD

then  $\overline{MN} \parallel \overline{AB}$ ,  $\overline{MN} \parallel \overline{DC}$ , and  $MN = 1/2(AB + CD)$



**Kite**

- a quadrilateral that has two pairs of consecutive congruent sides
- opposite sides are not congruent
- the congruent angles of a kite are formed by the noncongruent adjacent sides.

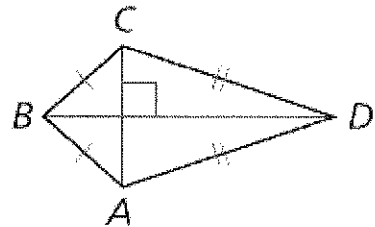


**Kite Diagonals Theorem**

- if a quadrilateral is a kite, then its diagonals are perpendicular

if quadrilateral ABCD is a kite

then  $\overline{AC} \perp \overline{BD}$

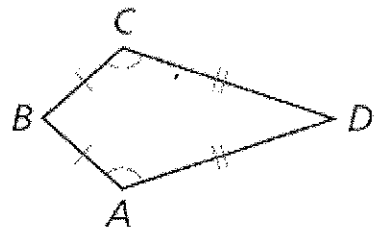


**Kite Opposite Angles Theorem**

- if a quadrilateral is a kite, then exactly one pair of opposite angles are congruent

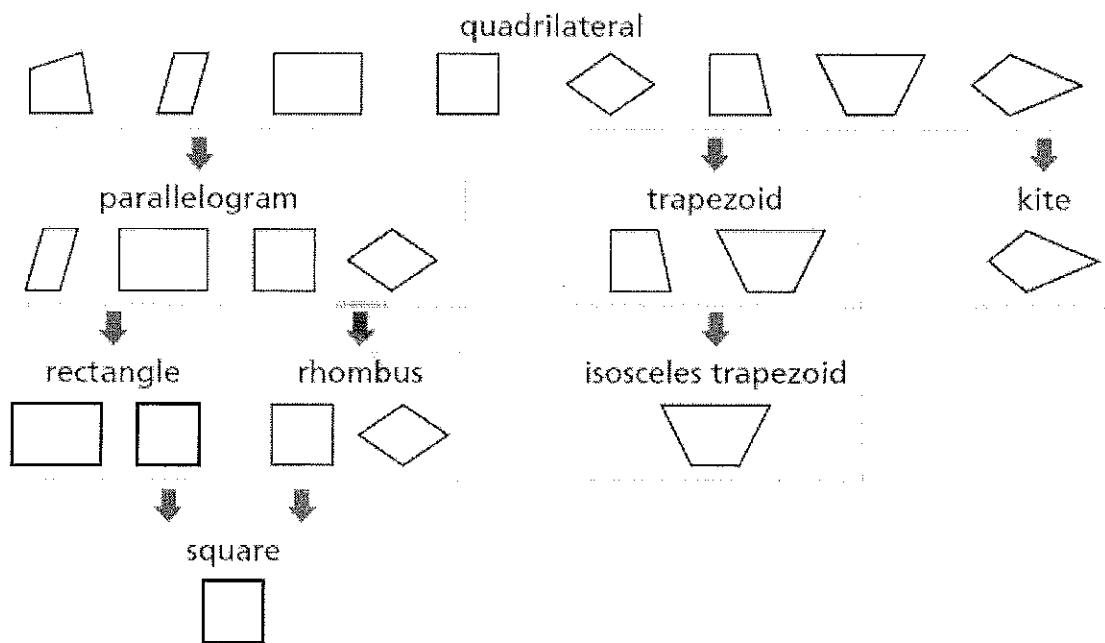
if quadrilateral ABCD is a kite and  $\overline{BC} \cong \overline{BA}$

then  $\angle A \cong \angle C$  and  $\angle B \cong \angle D$



**Identifying Special Quadrilaterals**

Each shape in the diagram has the properties of the shapes linked above it. For example, a rhombus has the properties of a parallelogram and a quadrilateral.



**Example 1:**

Prove that ABCD is a trapezoid and decide whether it is isosceles. Show all work.

Slope<sub>AB</sub> = \_\_\_\_\_

Slope<sub>BC</sub> = \_\_\_\_\_

Slope<sub>CD</sub> = \_\_\_\_\_

Slope<sub>AD</sub> = \_\_\_\_\_

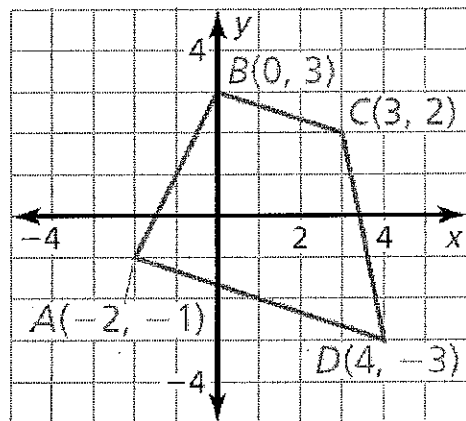
AB = \_\_\_\_\_

BC = \_\_\_\_\_

CD = \_\_\_\_\_

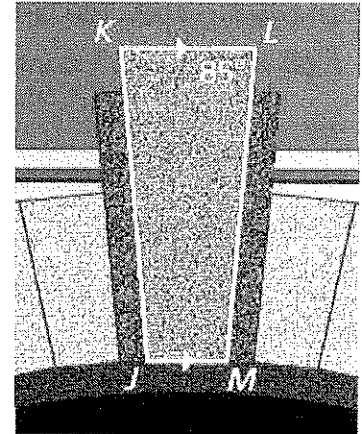
AD = \_\_\_\_\_

Isosceles? Yes/No \_\_\_\_\_



**Example 2: Using Properties of Isosceles Trapezoids**

The stone above the arch in the diagram is an isosceles trapezoid. Find  $m\angle K$ ,  $m\angle M$ , and  $m\angle J$ . Explain your reasoning.



$m\angle J =$  \_\_\_\_\_

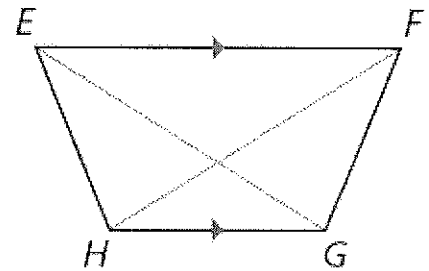
$m\angle K =$  \_\_\_\_\_

$m\angle M =$  \_\_\_\_\_

Reasoning:

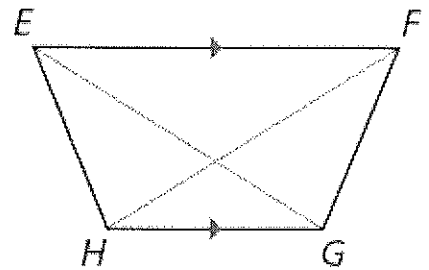
**Example 3:**

If  $EG = FH$ , is trapezoid EFGH isosceles? Yes/No  
Explain your reasoning:



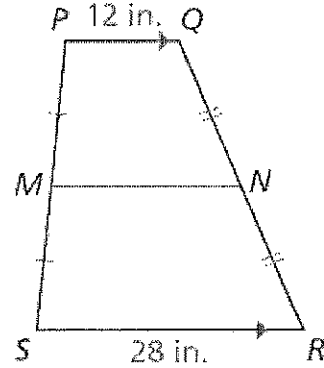
**Example 4:**

If  $m\angle HEF = 70^\circ$  and  $m\angle FGH = 110^\circ$ , is trapezoid EFGH isosceles? Yes/No  
Explain your reasoning:



**Example 5: Using the Midsegment of a Trapezoid**

In the diagram,  $\overline{MN}$  is the midsegment of trapezoid PQRS. Find MN. Show all work.



MN = \_\_\_\_\_

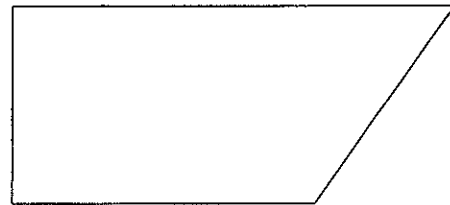
**Example 6:**

In trapezoid JKLM,  $\angle J$  and  $\angle M$  are right angles, and JK = 9 centimeters. The length of midsegment NP of trapezoid JKLM is 12 centimeters.

Using a ruler, label trapezoid JKLM and sketch its midsegment.

Find ML. Explain your reasoning.

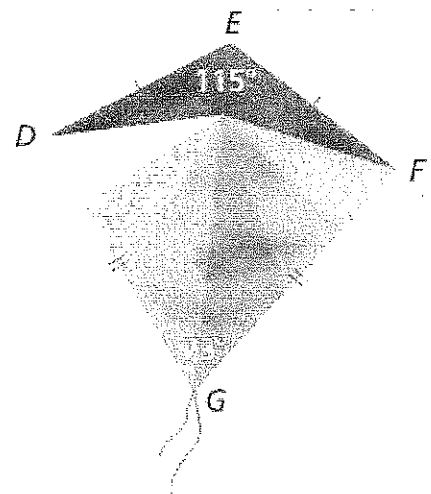
ML = \_\_\_\_\_



**Example 7: Finding Angle Measures in a Kite**

Find  $m\angle D$  in the kite shown. Show all work.

$m\angle D =$  \_\_\_\_\_



**Example 8:**

In a kite, the measures of the angles are  $3x^\circ$ ,  $75^\circ$ ,  $90^\circ$ , and  $120^\circ$ . Find the value of  $x$ .  
What are the measures of the angles that are congruent? Show all work.

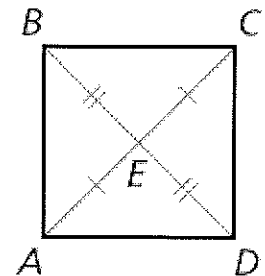
$$x = \underline{\hspace{2cm}}$$

$$\text{Congruent angle measures} = \underline{\hspace{2cm}}$$

**Example 9: Identifying a Quadrilateral**

What is the most specific name for quadrilateral ABCD?  
Explain your reasoning.

Hint: ABCD looks like a square. But you must rely **only** on marked information when you interpret a diagram.



Quadrilateral: \_\_\_\_\_

Reasoning:

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In a kite, the measures of the angles are  $3x^\circ$ ,  $75^\circ$ ,  $90^\circ$ , and  $120^\circ$ . Find the value of  $x$ .  
What are the measures of the angles that are congruent? Show all work.

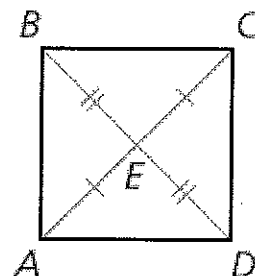
$$x = \underline{\hspace{2cm}}$$

$$\text{Congruent angle measures} = \underline{\hspace{2cm}}$$

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Quadrilateral: \_\_\_\_\_

Reasoning: